

Theory of Hall Anomaly in the Mixed State *

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Abstract

Analogous to Peierls' arguments for the 'anomalous' Hall in metals I demonstrate that the Hall anomaly in the mixed state of superconductors, the sign change of the Hall resistivity, is a property of a vortex many-body correlation, and show that the anomaly is due to the competition between vortex vacancies and interstitials. Within this vortex many-body effect picture, many features of the complicated Hall effect can be understood in quantitative terms. For example, the expression for the vacancy formation energy is obtained, the scaling relation between the Hall and longitudinal resistivities with the power varying between 1 and 2 is found to depend on sample details, and near the superconducting transition temperature and for small magnetic fields the Hall conductivity is proportional to the inverse of the magnetic field and to the quadratic of the difference between the measured and the transition temperatures.

In the past 10 years we have experienced the tremendous successes of the constructions and applications of vortex many-body theories [1], such as in the calculation of the longitudinal resistivity in the mixed state, a quantity vital for practical applications of superconductors. It is somehow surprising to notice that when one encounters the transverse

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resistivity, the Hall effect, theoretical models normally proposed are mostly of independent vortex dynamics type. It is now well documented that the Hall effect in the mixed state is very rich in behavior. Two of its features, the small Hall angle and the sign change of the Hall resistivity, are very noticeable and are the tests for theoretical models. They are usually referred to as the anomalous Hall effect, which for short I will call the Hall anomaly in this paper, although it may not be a proper name. Indeed, this Hall anomaly is ubiquitous below the superconducting transition temperature, occurring in the mixed state of both conventional and oxide superconductors. The crucial ingredient for the Hall effect is the transverse force, the Magnus force, on a vortex. This force plays the same role as the Lorentz force for electrons. Since a straightforward application of the Magnus force in any independent vortex dynamics model cannot explain this phenomenon, there are questions on the intrinsic nature of the transverse force in the vortex dynamic equation. This force has been subsequently modified into various forms [1], which again causes inconsistencies in theories. Since 60's we have the difficult task of constructing a consistent and falsifiable Hall anomaly theory.

This situation is identical to what had been encountered by Peierls in the late 20's on the usual Hall effect in solids: It has been known since earlier 20th century that the Hall effect in metals can be negative, positive, and zero, while the direct consequence of the Lorentz force in the Drude model predicts only the negative sign, corresponding to the charge of electrons in metals. This apparent contradiction was the famous anomalous Hall effect at that time. Peierls resolved this puzzle with the concept of the hole motion, by consideration of the competition between the electron many-body effect caused by Fermi statistics and the periodic lattice potential. Peierls's particle and hole concept has now become a fundamental building block of modern transport theory in solids, widely used in the semiconductor industry. Similar concept of holes in the filled electron sea (nowadays called positrons) was conceived by Dirac, and their discovery had won C.D. Anderson the Nobel prize. In his classical Hall effect papers [2], Peierls stressed that 'an explanation of this paradox was impossible as long as the electrons were visualised as freely-moving in the

metal', a statement applicable to the present Hall anomaly in the mixed state.

In the present paper using the analogy to the hole Hall effect in solids I attempt to solve the Hall anomaly in mixed state by showing that it can be understood based on the defect motions, vacancies and interstitials, in a pinned vortex lattice, and emphasize that the anomaly is a property of the many-body correlation rather than that of an individual vortex. There is a complete parallel comparison between the arguments for the Hall effect in solids and those in the mixed state, summarized in TABLE I. Comparing with the complex mixed state Hall phenomena, my theory is undoubtedly in its initial state. But it appears that the main physics has been captured. There are a few quantitatively testable predictions. I will demonstrate that vacancies can have the lowest energy scale, and they dominate the thermal activation contributions to the vortex motion at low temperatures. This causes the sign change of Hall effect. The almost equal numbers of vacancies and interstitials, together with their thermal activation type motions, and guided motions in the vortex lattice background, make the Hall angle small. The present theory is consistent with other measurement such as the Nernst effect. In the following I present arguments leading to the theory, and discuss its predictions and conditions. Although the theory has only been quantified for a few ideal cases and is necessarily crude at this stage, in the view of Peierls' solution to the older Hall anomaly I believe it paves the way for a complete theory. For simplicity, I will consider an isotropic s-pairing superconductor with one type of charge carriers in two dimension. In this situation vortices(or straight vortex lines) can be viewed as point particles.

(Put TABLE I here.)

Now I turn to the quantitative side of the present theory. The vortex dynamic equation for a j -th vortex in the sample takes the form of the Langevin equation identical to that of a charged particle in the presence of a magnetic field:

$$m\ddot{\mathbf{r}}_j = q\frac{\rho_s}{2}hd(\mathbf{v}_{s,t} - \dot{\mathbf{r}}_j) \times \hat{z} - \eta\dot{\mathbf{r}}_j + \mathbf{F}_p + \mathbf{f}, \quad (1)$$

with an effective mass m , a pinning force \mathbf{F}_p , a vortex viscosity η , and a fluctuating force \mathbf{f} . In Eq.(1) $q = \pm 1$ is the vorticity, h the Planck constant, ρ_s the superfluid electron number

density at temperature T , d the thickness of the superconductor film, and \hat{z} the unit vector in z -direction. If there is a temperature gradient, the thermal force $\mathbf{F}_T = -s_\phi \nabla T$ should be added in at the right hand of Eq.(1), with s_ϕ the entropy carried by a vortex. The term associated with the total superconducting electron velocity $\mathbf{v}_{s,t} = \mathbf{v}_s + \mathbf{v}_{s,in}$ and the vortex velocity $\dot{\mathbf{r}}$ at the right side of Eq.(1) is the transverse force. Although $\mathbf{v}_{s,t}$ is due to all other vortices, here I split it into two parts, with \mathbf{v}_s corresponding to the rearrangement of vortices due to the externally applied supercurrent and $\mathbf{v}_{s,in}$ accounting for the rest contribution describing the vortex interaction without external current. In the following I will assume \mathbf{v}_s is small such that this splitting is appropriate. It has been shown that under the general properties of a superconductor the transverse force must take such a form as a result of topological constraint. [3] The large amplitude of this force has been verified by a recent experiment [4].

It is a common knowledge that in the mixed state of any real superconductor the many-body correlation between vortices and the pinning effect cannot be ignored. The competition between them is the source of the rich static and dynamical properties of flux phases [1]. However, almost all previous models attempted to solve the Hall anomaly have ignored this strong many-body correlation, which gives rise to the Abrikosov lattice if fluctuations were ignored. If there were no pinning for vortices, the whole vortex lattice would move together under the influence of an externally applied current in the same manner as that of independent vortices. Hence one would get the same sign of the Hall resistivity in both superconducting and normal states. In the presence of pinning centers the vortex lattice will be pinned down. In such a situation the motion of the vortex lattice is made possible by various kinds of defect motions caused by thermal fluctuations. I will argue below that at low temperatures vacancies can dominate the contribution to the motion, and the Hall anomaly occurs.

Let me discuss first the energy scales in the problem. To simplify the presentation, only thin films will be discussed except otherwise specified. The obvious energy scale will be the depinning of a single vortex. This energy is $\epsilon_0 \equiv d(\Phi_0/4\pi\lambda_L)^2$, determining by the

core energy. Here $\Phi_0 = hc/2e$ is the flux quantum, and $\lambda_L^2 = m^*c^2/8\pi\rho_se^2$ the London penetration depth, and m^* the effective mass of a Cooper pair. This energy scale also sets the scale for the the creations of a vortex, an antivortex, and a vortex pair. One of the simplest collective excitations is a dislocation pair. Its energy scale is $(\epsilon_0/2\sqrt{3}\pi)\ln(r/a_0)$ with r the distance between dislocations and a_0 the vortex lattice constant. Apart from a logarithmic factor, this energy scale is about 10 times smaller than single vortex energy scale ϵ_0 . It is then energetically more favorable to have dislocation pairs in the lattice comparing. For temperature $T \ll \epsilon_0$ I can ignore the contribution from the vortices hopping out of pinning centers and the creation of vortex-antivortex pairs. The vortex lattice is then effectively pinned down. The smallest energy scale will correspond to point-like defects, vacancies and interstitials. If viewing them as the smallest dislocation pairs, I immediately have the estimated energy scale for their formation energies as, by putting $r \sim 2a_0$,

$$\epsilon_v \sim \frac{1}{2\sqrt{3}\pi} \left(\frac{\Phi_0}{4\pi\lambda_L} \right)^2 d. \quad (2)$$

This result is valid for an intermediate magnetic field B : $H_{c1} < B < H_{c2}/2$. The linear film thickness d dependence of the vacancy formation energy ϵ_v holds for thin enough films, and independent of the thickness for thicker ones due to the z -direction correlation, in which the straight line assumption in reaching Eq.(2) is no longer valid. I do not have a quantitative theory for the crossover thickness yet, but in thin enough films Eq.(2) has been verified. [5] It is clear from the above analysis that vacancies and interstitials have the lowest excitation energy scales.

Based on the general observations for the defect formations in crystals and the theoretical and experimental evidences in vortex lattices I argue now that the vacancy formation energy is even lower than that of an interstitial for a wide parameter regime. The experimental observations at low magnetic fields have shown the abundance of vacancies comparing with interstitials. The natural explanation is that the the vacancy formation energy is lower than that of interstitials, therefore by thermal fluctuations vacancies have a higher density. The theoretical understanding is that the lower vacancy formation energy is a result of the

strong short range and repulsive nature of the interaction between vortices at low magnetic fields, as have been shown. Similar phenomenon has also been observed in other crystalline structures. At high magnetic fields the vortex interaction becomes of long range comparing with the vortex lattice constant. The short range behavior which determines the formation energy difference between vacancies and interstitials becomes relatively unimportant. The difference between formation energies of vacancy and interstitial will be smaller in this limit. Overall, vacancies have a lower formation energy for a finite range interaction. I conclude that they will dominate thermal fluctuation contributions to resistivities at low enough temperatures. [5]

Using the analogy to the dynamics of a hole or a particle in a solid in the presence of a magnetic field, with a pinned perfect vortex lattice as a filled valence band and a vacancy in real space as a hole in the energy space, illustrated in the TABLE I, the desired transverse force on a defect, vacancy or interstitial, can be obtained as

$$\mathbf{F}_M^d = \mp q \frac{\rho_s}{2} h d (\mathbf{v}_s - \dot{\mathbf{r}}_0) \times \hat{z}. \quad (3)$$

This equation shows that both a vacancy and an interstitial will move along the direction of the applied supercurrent \mathbf{v}_s . This implies that vortices defining vacancies move against the direction of \mathbf{v}_s , a result of the many-body correlation and pinning. This leads me to the main conclusion that at low enough temperatures the sign of the Hall resistivity is different from its sign in the normal state because of the dominance of vacancies. Quantitatively, vacancies and interstitials may be considered as independent particles moving in the periodic potential formed by the vortex lattice and a random potential due to the residue effect of pinnings. The potential height of the periodic potential as well as that of the random potential is an order of ϵ_v . Assuming the vacancy (interstitial) density $n_v(n_i)$ in a steady state, the longitudinal resistivity is

$$\rho_{xx} = \frac{h}{2e^2} \sum_{l=v,i} \frac{\eta_l \rho_s d h / 2}{\eta_l^2 + (\rho_s d h / 2)^2} \frac{n_l}{\rho_s}, \quad (4)$$

and the Hall resistivity

$$\rho_{yx} = \frac{h}{2e^2} \sum_{l=v,i} q_l \frac{(\rho_s dh/2)^2}{\eta_l^2 + (\rho_s dh/2)^2} \frac{n_l}{\rho_s}, \quad (5)$$

with $q_v = -q$ and $q_i = q$. Here $\eta_{v,i}$ are the effective vacancy and interstitial viscosities, related to their diffusion constants in the periodic potential due to the vortex lattice by the Einstein relation between the diffusion constant and the mobility. It should be pointed out that contributions of other vortex motions to resistivities such as vortex-antivortex pairs, which are omitted here for their smaller activation probabilities, are additive to those of vacancies, and that the including of the normal fluid (quasiparticle) contributions is straightforward [6].

Under the driving of a temperature gradient the effective thermal force felt by a vacancy is opposite in sign to the force felt by an interstitial or a vortex in direction but equal in magnitude, $\mathbf{F}_T^v = +s_\phi \nabla T$. Then the Nernst effect due to vacancies has the same sign as that of vortices or interstitials. Therefore my theory gives that in the Hall anomaly regime there is no sign change for the Nernst effect, and furthermore, the Nernst effect is more pronounced because of the large contribution due to both vacancies and interstitials. This is in agreement with the experimental observations [6].

There are several qualitative implications of the present theory worth further discussions. In the above physical picture, to obtain a maximum contribution of vacancies, I need the vortex lattice to define vacancies and a sufficiently strong pinnings to prevent the sliding of vortex lattice to obtain a maximum contribution of vacancies. The existence of a whole lattice structure is nevertheless unnecessary. Sufficiently large local crystalline structures, like lattice domains, will be enough to define vacancies. Therefore vacancy-like excitations in a vortex liquid state can exist, because of the existence of large local orderings. Whether or not this is also true for a vortex glass state depending on details. For example, a further lowering of temperature may quench a vortex system into a glass state with no local crystalline structure. The usual counting of defects does not work. However, a general classification based on the Delauney triangulation method can still be applied [7], and the careful analysis of the defect stabilities and relative energy scales as well as their mobilities remains to be

done. It is possible that the sign of the Hall resistivity will change again in this parameter regime. On the other hand, for a fixed temperature if the pinning is too strong, for example, the (random) pinning center density is much larger than the vortex density, vortices will be individually pinned down and the local lattice structure required for the formations of vacancies and interstitials will be lost. This suggests that the Hall anomaly only exists in a suitable range of pinnings and magnetic fields, that is, for $B_l < |B| < B_u$ with the lower and upper critical fields determined by pinning as well as by temperature.

To make a contact with experiments I work out several limiting cases of Eqs.(4,5) below. At low temperatures the motions of vacancies and interstitials in the vortex lattice are thermal hoppings: $\eta_{v,i} = \eta_0 e^{a_{v,i} \epsilon_v / K_B T}$ and $n_{v,i} = n_0 e^{b_{v,i} \epsilon_v / K_B T}$. If $\eta_v, \eta_i \ll \rho_s dh / 2$, corresponding to the Hall angle $|\tan \theta| = |\rho_{yx} / \rho_{xx}| \ll 1$ common in experiments,

$$\tan \theta = -q \frac{\rho_s dh}{2\eta_0} \frac{e^{-(2a_v+b_v) \frac{\epsilon_v}{k_B T}} - e^{-(2a_i+b_i) \frac{\epsilon_v}{k_B T}}}{e^{-(a_v+b_v) \frac{\epsilon_v}{k_B T}} + e^{-(a_i+b_i) \frac{\epsilon_v}{k_B T}}}.$$

For two temperature limits,

$$\tan \theta = \begin{cases} -q \frac{\rho_s dh}{2\eta_0} \frac{\gamma}{2} \frac{\epsilon_v}{k_B T}, & k_B T \geq \epsilon_v. \\ -q \frac{\rho_s dh}{2\eta_0} e^{-a_v \frac{\epsilon_v}{k_B T}}, & k_B T < \min\{1, \gamma\} \epsilon_v. \end{cases} \quad (6)$$

Here $\gamma = 2a_i + b_i - 2a_v - b_v$ is a numerical factor. The high temperature limit $k_B T \geq \epsilon_v$ is achieved near superconducting transition temperature T_{c0} , but the thermal creation of a vortex-antivortex pair is still relatively improbable, because the relevant energy scale ϵ_0 is about 10 times bigger than ϵ_v . In the low temperature limit both longitudinal and Hall resistivities vanish exponentially. A scaling relation between them is

$$\rho_{yx} = A \rho_{xx}^\nu, \quad 1 \leq \nu \leq 2 \quad (7)$$

with $A = -q(\rho_s dh / 2\eta_0)^{b_v / (a_v + b_v)} (2e^2 \rho_s / h n_0)^{a_v / (a_v + b_v)}$, and the power $\nu = (2a_v + b_v) / (a_v + b_v)$ varying between 1 and 2, depending on the detail of a sample which determines the numerical factors a_v and b_v . For example, if all vacancies are produced by pinnings, $b_v = 0$ and $\nu = 2$. In the other limit, if all vacancies are produced by thermal activations, and if $a_v \ll 1$, $b_v = 1$ and $\nu \simeq 1$.

Another useful quantity is the Hall conductivity $\sigma_{xy} = \rho_{yx}/(\rho_{xx}^2 + \rho_{yy}^2)$. Under the same assumption of $\eta_v, \eta_i \ll \rho_s dh/2$ I obtain the Hall conductivity due to vacancies and interstitials, from Eqs.(4,5), as

$$\sigma_{xy} = -q \frac{2e^2}{h} \frac{\rho_s}{n_0} \frac{e^{-(2a_v+b_v)\frac{\epsilon_v}{k_B T}} - e^{-(2a_i+b_i)\frac{\epsilon_v}{k_B T}}}{\left[e^{-(a_v+b_v)\frac{\epsilon_v}{k_B T}} + e^{-(a_i+b_i)\frac{\epsilon_v}{k_B T}} \right]^2}.$$

Again, in two temperature limits,

$$\sigma_{xy} = \begin{cases} -q \frac{2e^2}{h} \frac{\rho_s}{n_0} \frac{\gamma}{4} \frac{\epsilon_v}{k_B T}, & k_B T \geq \epsilon_v. \\ -q \frac{2e^2}{h} \frac{\rho_s}{n_0} e^{+b_v \frac{\epsilon_v}{k_B T}}, & k_B T < \min\{1, \gamma\} \epsilon_v. \end{cases} \quad (8)$$

Here $0 \leq b_v \leq 1$ and $\gamma \sim O(1)$. Near the superconducting transition temperature T_{c0} , $\rho_s = \rho_{s0}(1 - T/T_{c0})$ and $\epsilon_v = \epsilon_{v0}(1 - T/T_{c0})$ because of the London penetration depth in Eq.(2). Making a further assumption of $n_0 = B/\Phi_0$, with Φ_0 the flux quantum, from Eq.(8) I obtain

$$\sigma_{xy} = \alpha_1 \frac{(1 - T/T_{c0})^2}{B}, \quad (9)$$

with $\alpha_1 = -q(2e^2/h)\rho_{s0}\Phi_0\gamma\epsilon_{v0}/4k_B T_{c0}$. Taking $\rho_{s0} = 10^{21}/cm^3$, $\gamma = 1$, and $\epsilon_{v0}/k_B T_{c0} = 50$, $|\alpha_1| \sim 20 T \mu\Omega^{-1} cm^{-1}$.

Two comments are in order. 1. Since vortex interaction terms cancel each other when summing over all vortices, one may conclude that there is no many-body correlation effect on the sign of the Hall resistivity. This absence of Hall anomaly is the result of the underestimation of the many-body correlation, when applied to electrons directly contradict to both Peierls' theory and experiments. 2. One may resort the motion of an antivortex in a vortex-antivortex pair to explain the anomaly. While it is a physically possible scenario, as discussed above the creation energy of an antivortex is about 10 times larger than that of a vacancy, which makes it energetically unfavorable at a relatively low temperature. Furthermore, since an antivortex feels the same thermal force as a vortex, $\mathbf{F}_T = -s_\phi \nabla T$, it has an opposite sign contribution to that of a vortex for the Nernst effect, in conflicting with experiments. [5,6]

Finally, let me summarize the main features of the present theory. Like the same effect in solids, Hall effect in the mixed state is a many-body phenomenon. It is impossible to give it a consistent explanation by any independent vortex dynamics model, as long ago pointed out by Peierls in a closely related situation. The time reverse symmetry of vortex dynamics is broken by the transverse force, or the Magnus force. Together with the broken vortex vacancy-interstitial symmetry finite Hall effect appears. The dominant role played by defects shows that fluctuations are essential in the present theory. Although it has several quantitative predictions to be tested, the decisive one may be the combination of the real space-time observation of the vortex vacancies and the Hall measurement.

Acknowledgments

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TABLE I. Qualitative Physical Picture

two major conclusions:

1. Many-body correlation responsible for the Hall effect;
2. No one-one correspondence between the Hall effect and the transverse force.

	MIXED STATE	SOLIDS
Fundamental particles	Vortices	Electrons
Many-body ground states	Pinned vortex lattice	Filled valence band
Excitations, I	Vortex interstitials	Particles
Excitations, II	Vortex vacancies	Holes
Symmetry condition, I	Interstitial-vacancy asymmetry	Particle-hole asymmetry
Symmetry condition, II	Transverse force(Magnus force)	Lorentz force